

Maths

Frequency Distribution

We divide data into classes

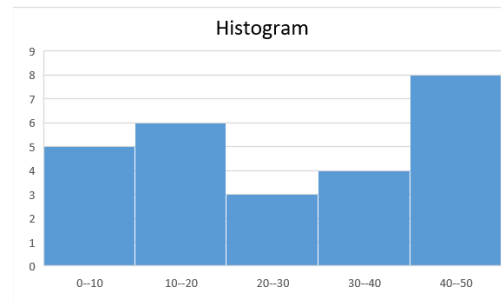
Data: 3, 4, 4, 12, 15, 31, 32, 36, 38, 41, 45, 46, 46, 37, 39

Class(x)	0-10	10-20	20-30	30-40	40-50
Frequency(f)	3	2	1	6	4

Can be turned to :

Histogram (Bar Chart)

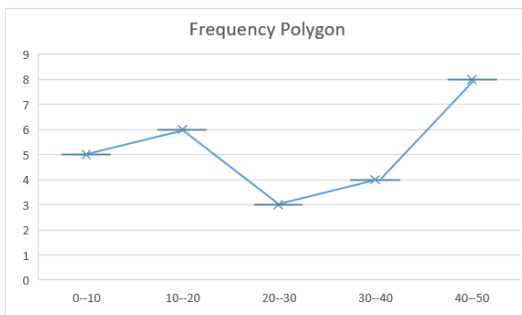
- 1- X-axis represents classes and must be continuous (if a class frequency it's =0)
- 2- Y-axis represents frequency , we can scale it by dividing by largest frequency.



has no
the

Frequency Polygon (joined lines)

We connect between the midpoints of every bar



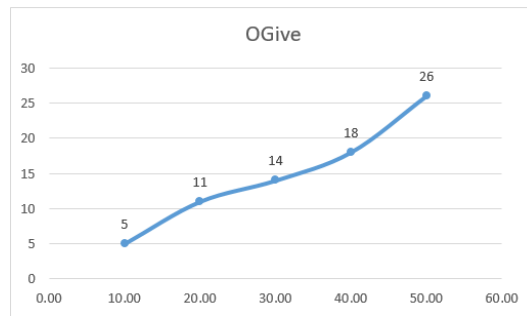
Cumulative frequency polygons (OGIVE)

We make a new table of x_i and y_i

x_i : is the largest value in the class border

y_i : add the class's frequency to the frequency of the previous ones

x_i	10	20	30	40	50
y_i	3	5	6	12	16

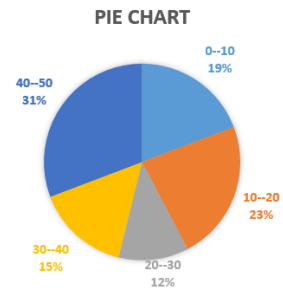


Circle (Pie chart)

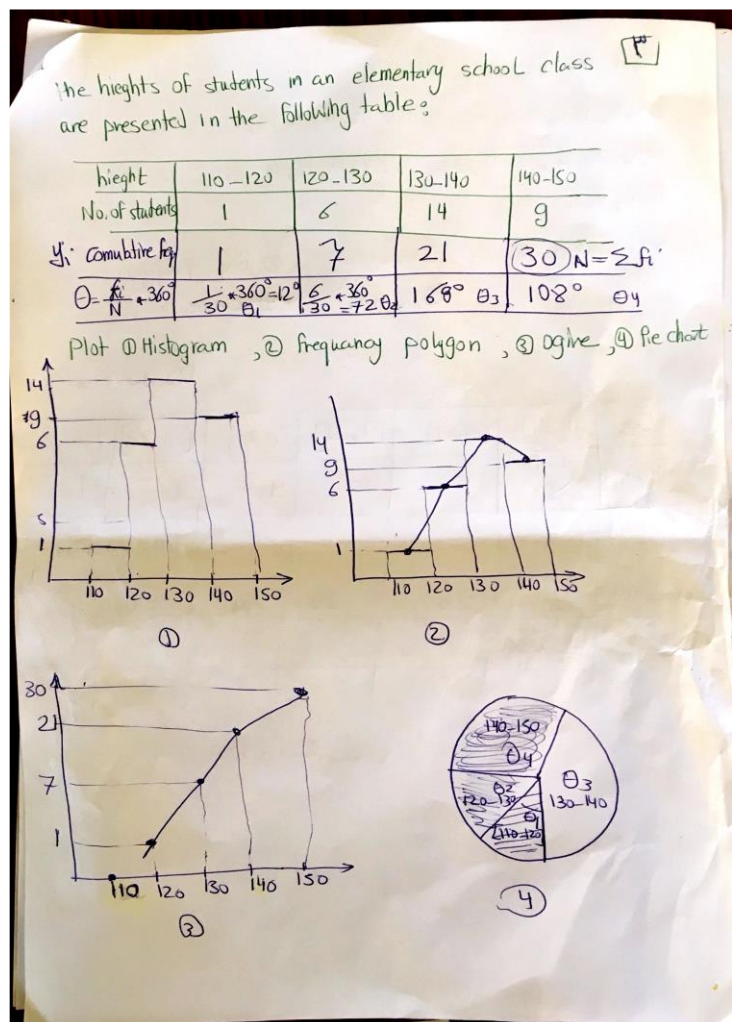
We divide the circle into segments , every segment's angle depends on it's frequency according to

$$\theta_i = \frac{f_i}{N} (2\pi) = \frac{f_i}{N} (360)$$

$$N = \sum f_i$$



Good example on how to solve :



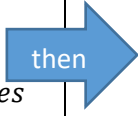
Data properties

For any data I can get :

(1) Average { Mean
Median
Mode

(2) Standard Deviation S.D σ

1-1 Mean

With repeat	Without repeat	Short-cut method
$\text{Mean}(\mu) = \frac{(\sum f_i x_i)}{\sum f_i}$ <p>x_i: is the midpoint of class period if class is a number then it is x_i</p>	$a = \frac{\sum x_i}{n}$ <p>a : mean of centres of classes</p>	<div style="text-align: center;">  </div> $\mu = a + \frac{\sum f_i d_i}{N}$ <p>$d_i = x_i - a$ a : any number</p>

2 Standard Deviation σ

	Shortest method
$\sigma = \sqrt{\left(\frac{\sum f_i (x_i - \mu)^2}{\sum f_i} \right)}$	$\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2}$

Median

For normal set of numbers

n(odd)	n(even)
$\frac{1}{2}(n + 1)$	$\frac{1}{2}n$, $\frac{1}{2}(n + 1)$

For a table

We get $\frac{\sum f_i}{2}$ and search for it in the cumulative column then the class is the median class

After determining the median class (looking at cumulative) then,

$$\text{median} = L + \frac{\frac{\sum f_i}{2} - C}{f} * i$$

L : lower limit of median's class

f : frequency of median class

C : cumulative frequency of **previous** class

i : width of median class

$N : \sum f_i$

Mode

We determine its place in the class with largest frequency

$$\text{Mode} = L + \frac{f - f_{-1}}{2f - f_{-1} - f_1} * i$$

L : lower limit of mode class

f : frequency of mode class

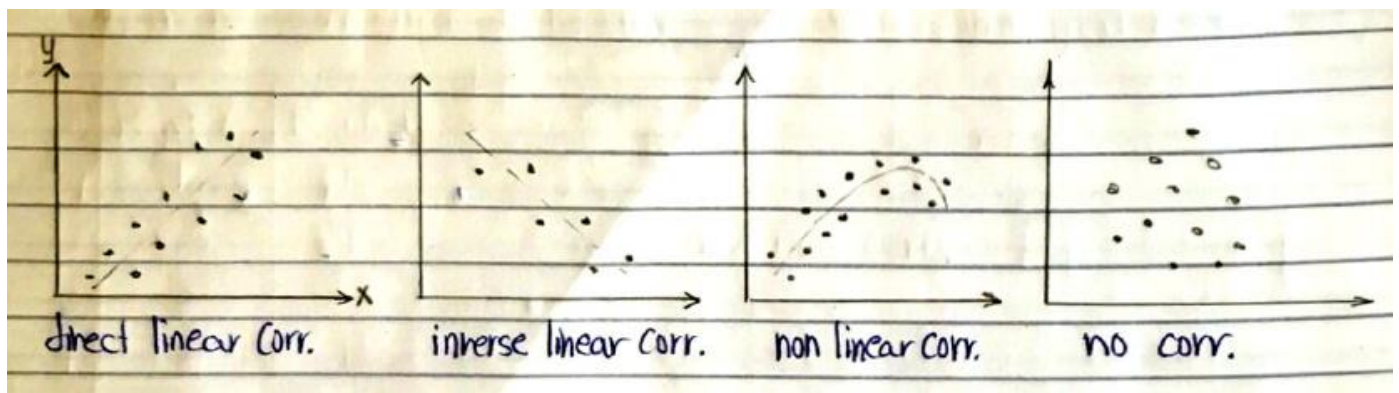
f_{-1} :: ~ ~ before

f_1 : ~ ~ after

i : width of mode class

Scatter diagram

To Show points (x,y) on a rectangular coordinate system



Correlation coefficient

Quantitative (قيم عددية)	Quantitative ; Qualitative بيانات وصفية - او قيم تدل على الترتيب
Linear , Pearson " r "	Rank , Spearman " ρ "
$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \sqrt{n\sum y^2 - (\sum y)^2}}$	$\rho = 1 - \left[\frac{6\sum d^2}{n(n^2 - 1)} \right]$ (d : rank x - rank y)

Type of correlation

$-1 \leq \text{corr. coeff.} \leq 1$				
1	~0.5	0	+ve	-ve
Complete corr	Medium	No corr	direct	inverse
	<div>> 0.5 Strong</div> <div>< 0.5 Weak</div>			

Prediction; Regression

Get prediction equation (relation between x,y)

Linear predict equation $y = a + bx$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$a = \bar{y} - b\bar{x}$$

$$\bar{x} = \frac{\sum x}{n} ; \bar{y} = \frac{\sum y}{n}$$

from that equation we can get value of any y as x known

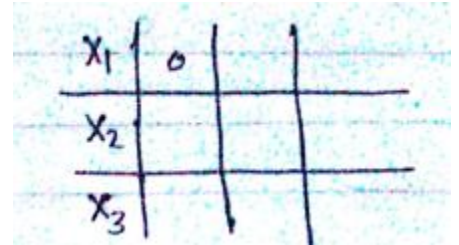
Multiple Correlation

We get r for every pair

$$r_{12} = \frac{n\sum x_1 x_2 - \sum x_1 \sum x_2}{\sqrt{n\sum x_1^2 - (\sum x_1)^2} \sqrt{n\sum x_2^2 - (\sum x_2)^2}}$$

$$r_{13} = \frac{n\sum x_1 x_3 - \sum x_1 \sum x_3}{\sqrt{n\sum x_1^2 - (\sum x_1)^2} \sqrt{n\sum x_3^2 - (\sum x_3)^2}}$$

$$r_{23} = \frac{n\sum x_2 x_3 - \sum x_2 \sum x_3}{\sqrt{n\sum x_2^2 - (\sum x_2)^2} \sqrt{n\sum x_3^2 - (\sum x_3)^2}}$$



$$x_1 = \beta_1 + \beta_2 x_2 + \beta_3 x_3$$

To get $\beta_1, \beta_2, \beta_3$ we have 2 methods

method (1)

$$x_1 = \beta_1 + \beta_2 x_2 + \beta_3 x_3$$

* \sum

$$\sum x_1 = n\beta_1 + \beta_2 \sum x_2 + \beta_3 \sum x_3$$

(1)

* $x_2 \rightarrow$ * \sum

$$\sum x_1 x_2 = \beta_1 \sum x_2 + \beta_2 \sum x_2^2 + \beta_3 \sum x_2 x_3$$

(2)

* $x_3 \rightarrow$ * \sum

$$\sum x_1 x_3 = \beta_1 \sum x_3 + \beta_2 \sum x_2 x_3 + \beta_3 \sum x_3^2$$

(3)

We solve 1,2,3 together to get $\beta_1, \beta_2, \beta_3$

method (2)

$$\sigma_1 = \sqrt{\frac{\sum x_1^2}{n} - \left(\frac{\sum x_1}{n}\right)^2}$$

$$\sigma_2 = \sqrt{\frac{\sum x_2^2}{n} - \left(\frac{\sum x_2}{n}\right)^2}$$

$$\sigma_3 = \sqrt{\frac{\sum x_3^2}{n} - \left(\frac{\sum x_3}{n}\right)^2}$$

$$\text{Note : } \frac{\sum x_1}{n} = \bar{X}_1 = \mu_{x_1}$$

$$x_1 = \beta_2 x_2 + \beta_3 x_3$$

$$\beta_2 = \frac{\sigma_1}{\sigma_2} \left(\frac{r_{12} - r_{13}r_{23}}{1 - r_{23}^2} \right)$$

$$\beta_3 = \frac{\sigma_1}{\sigma_3} \left(\frac{r_{13} - r_{12}r_{23}}{1 - r_{23}^2} \right)$$

Proofs

Show that for any value of a if $d_i = x_i - a$ that $\mu = a + \frac{\sum f_i d_i}{\sum f_i}$

$$\mu = \frac{\sum x_i f_i}{\sum f_i} \quad \text{but } d_i = x_i - a \rightarrow x_i = d_i + a$$

$$\mu = \frac{\sum (d_i + a) f_i}{\sum f_i} = \frac{\sum f_i d_i}{\sum f_i} + \frac{\sum f_i a}{\sum f_i} = \frac{\sum f_i d_i}{\sum f_i} + a \frac{\sum f_i}{\sum f_i} = a + \frac{\sum f_i d_i}{\sum f_i}$$

shortest method , prove $\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{\sum f_i}} \quad \text{put : } \mu = \frac{\sum x_i f_i}{\sum f_i}$$

$$\sigma^2 = \frac{1}{\sum f_i} \sum f_i [x_i^2 - 2\mu x_i + \mu^2] = \frac{1}{\sum f_i} \left[\sum f_i x_i^2 - \frac{2(\sum f_i x_i)^2}{\sum f_i} + \frac{(\sum f_i x_i)^2 \sum f_i}{(\sum f_i)^2} \right]$$

$$\sigma^2 = \frac{1}{\sum f_i} \left[\sum f_i x_i^2 - \frac{\sum f_i x_i^2}{\sum f_i} \right]$$

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$$

show that for any value a $\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$

$$d_i = x_i - a$$

$$\sum f_i = N$$

$$\sigma = \frac{1}{N} \sum f_i [(x_i - a) - (\mu - a)]^2 = \frac{1}{N} \sum f_i (d_i - (\mu - a))^2$$

$$= \frac{1}{N} \left[\sum f_i d_i^2 - 2(\mu - a) \sum f_i d_i + (\mu - a)^2 \frac{\sum f_i}{N} \right]$$

$$\mu = a + \frac{\sum f_i d_i}{N}$$

$$\sigma = \frac{1}{N} \left[\sum f_i d_i^2 - 2 \frac{(\sum f_i d_i)^2}{N} + \frac{(\sum f_i d_i)^2}{N} \right]$$

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$$

let μ_x and μ_y be means of x_i and y_i , $X = x - \mu_x$, $Y = y - \mu_y$ show that

$$r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})} = \frac{\sum XY}{n\sigma_x\sigma_y} = \frac{\text{cov}(x, y)}{\sigma_x\sigma_y}$$

where σ_x , σ_y are standard deviation of x_i and y_i

$$f_i = 1, \sum f_i = n$$

$$\sigma_x^2 = \frac{\sum f_i (x - \mu_x)^2}{\sum f_i} = \frac{1}{n} \sum (x - \mu_x)^2$$

$$\sigma_x^2 = \frac{1}{n} \sum X^2 \quad \sigma_y^2 = \frac{1}{n} \sum Y^2$$

$$\sum XY = \sum (x - \mu_x)(y - \mu_y) = \sum xy - \mu_y \sum x - \mu_x \sum y + \mu_x \mu_y \sum 1 =$$

$$\mu_x = \frac{\sum f_i x_i}{\sum f_i} = \frac{1}{n} \sum x \quad ; \mu_y = \frac{1}{n} \sum y$$

$$\sum XY = \sum xy - \frac{\sum y}{n} \sum x - \frac{1}{n} \sum x \sum y + \frac{1}{n} \sum x \sum y$$

$$\therefore \sum XY = \sum xy - \frac{1}{n} \sum x \sum y \rightarrow (1)$$

$$\sum X^2 = \sum (x - \mu_x)^2 = \sum x^2 - 2\mu_x \sum x + \mu_x^2 \sum 1$$

$$\sum X^2 = \sum x^2 - 2n\mu_x^2 + n\mu_x^2 = \sum x^2 - n\mu_x^2 = \sum x^2 - n\left(\frac{\sum x}{n}\right)^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$\therefore \sqrt{\sum X^2} = \sqrt{\frac{1}{n} [n\sum x^2 - (\sum x)^2]}$$

$$\therefore \sqrt{\sum Y^2} = \sqrt{\frac{1}{n} [n\sum y^2 - (\sum y)^2]}$$

$$\text{then : } r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})} = \frac{\sum xy - \frac{1}{n} \sum x \sum y}{\left(\sqrt{\frac{1}{n} [n\sum x^2 - (\sum x)^2]}\right) \left(\sqrt{\frac{1}{n} [n\sum y^2 - (\sum y)^2]}\right)}$$

$$\frac{\sum xy - \frac{1}{n} \sum x \sum y}{\frac{1}{n} \left(\sqrt{[n\sum x^2 - (\sum x)^2]}\right) \left(\sqrt{[n\sum y^2 - (\sum y)^2]}\right)} = \frac{n\sum xy - \sum x \sum y}{\left(\sqrt{[n\sum x^2 - (\sum x)^2]}\right) \left(\sqrt{[n\sum y^2 - (\sum y)^2]}\right)} \quad \#1$$

$$\sigma_x^2 = \frac{1}{n} \sum X^2 \quad ; \quad \sigma_y^2 = \frac{1}{n} \sum Y^2$$

$$r = \frac{\sum XY}{\sqrt{n} \sigma_x \sqrt{n} \sigma_y} = \frac{\sum XY}{n \sigma_x \sigma_y} = \frac{\text{cov}(x, y)}{n \sigma_x \sigma_y} \quad \#2, 3$$

$$\text{show that } \sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2r\sigma_x\sigma_y$$

$$\sigma_x = \sqrt{\frac{\sum f(x - \mu_x)^2}{\sum f}} \quad ; \quad \sigma_y = \sqrt{\frac{\sum f(y - \mu_y)^2}{\sum f}}$$

$$\sigma_x^2 = \frac{1}{n} \sum (x - \mu_x)^2 = \frac{1}{n} [\sum x^2 - 2\mu_x \sum x + n\mu_x^2]$$

$$\sigma_y^2 = \frac{1}{n} [\sum y^2 - 2\mu_y \sum y + n\mu_y^2]$$

$$r = \frac{\sum (x - \mu_x)(y - \mu_y)}{n\sigma_x\sigma_y}$$

$$\text{L. H. S} = \sigma_{x-y}^2 = \frac{1}{n} \sum [(x - y) - \mu_{x-y}]^2$$

$$\mu_{x-y} = \frac{\sum (x - y)}{n} = \frac{\sum x}{n} - \frac{\sum y}{n}$$

$$\mu_{x-y} = \mu_x - \mu_y$$

$$\sigma_{x-y}^2 = \frac{1}{n} \sum [(x - \mu_x) - (y - \mu_y)]^2 = \frac{1}{n} \sum [(x - \mu_x) - (y - \mu_y)]^2$$

$$= \frac{1}{n} \sum [(x - \mu_x)^2 - 2(x - \mu_x)(y - \mu_y) + (y - \mu_y)^2]$$

$$= \sigma_x^2 - 2r\sigma_x\sigma_y + \sigma_y^2 \quad \#$$

$$\text{let } x_i, y_i \text{ be ranked individual distinct data then } r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

$$\text{use : } d_i = x_i - y_i, \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} ; X = x - \mu_x, Y = y - \mu_y; r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})}$$

$$\text{The mean of Rank } x = \frac{1 + 2 + 3 + \dots + n}{n}$$

$$\text{Sum of arithmetic series} = \frac{n}{2} (a_1 + a_n)$$

so,

$$\mu_{R_x} = \frac{1}{n} \left[\frac{n}{2} (1 + n) \right] = \frac{n+1}{2}$$

$$\mu_{R_y} = \frac{n+1}{2}$$

$$d_i = X_i - Y_i = (x - \mu_{R_1}) - (y - \mu_{R_2}) = x_i - y_i$$

ΣX^2 بدلالة الترتيب r نحاول ايجاد كل جزء في ال

$$\Sigma X^2 = \Sigma (x - \mu_{R_1})^2 = \sum_{i=1}^n \left[x_i - \frac{n+1}{2} \right]^2$$

$$\Sigma X^2 = \Sigma x^2 - (n+1)\Sigma x + \left(\frac{n+1}{2} \right)^2 \Sigma 1 \rightarrow (1)$$

$$\Sigma x^2 = 1^2 + 2^2 + \dots + n^2 = \frac{(n(n+1)(2n+1))}{6}$$

$$\text{same, } \Sigma y^2 = \frac{(n(n+1)(2n+1))}{6}$$

$$\Sigma x = 1 + 2 + \dots + n = \frac{n}{2} [1 + n]$$

$$\text{same, } \Sigma y = 1 + 2 + \dots + n = \frac{n}{2} [1 + n]$$

subs in (1)

$$\begin{aligned} \Sigma X^2 &= \Sigma x^2 - (n+1)\Sigma x + \left(\frac{n+1}{2} \right)^2 \Sigma 1 \\ &= \frac{(n(n+1)(2n+1))}{6} - \frac{n}{2} (n+1)^2 + \left(\frac{n+1}{2} \right)^2 n \\ &= \frac{n(n+1)}{2} \left[\frac{2n+1}{3} - n - 1 + \frac{n+1}{2} \right] = \frac{n(n+1)}{2} \left[\frac{4n+2-6n-6+3n+3}{6} \right] \\ &= \frac{n(n+1)}{2} \left[\frac{n-1}{6} \right] = \frac{n(n^2-1)}{12} \\ \Sigma X^2 &= \frac{n(n^2-1)}{12} \end{aligned}$$

$$\Sigma Y^2 = \frac{n(n^2-1)}{12}$$

$$\Sigma XY = \Sigma (x - \mu_{R_1})(y - \mu_{R_2})$$

$$\Sigma d^2 = \Sigma (X - Y)^2 = \Sigma X^2 - 2\Sigma XY + \Sigma Y^2$$

$$\Sigma XY = \frac{1}{2} [\Sigma X^2 + \Sigma Y^2 - \Sigma d^2] = \frac{1}{2} \left[\frac{n(n^2-1)}{6} - \Sigma d^2 \right]$$

subs in r

$$r = \frac{\Sigma XY}{(\sqrt{\Sigma X^2})(\sqrt{\Sigma Y^2})} = \frac{\frac{1}{2} \left[\frac{n(n^2-1)}{6} - \Sigma d^2 \right]}{\frac{n(n^2-1)}{12}} = 1 - \frac{6\Sigma d^2}{n(n^2-1)} \quad \#$$

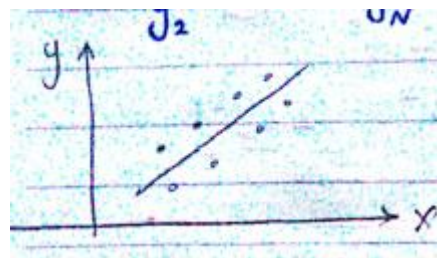
Get the value of a, b in regression line $y = a + bx$

$$y_1 = a + bx_1$$

$$y_2 = a + bx_2$$

.

$$y_n = a + bx_n$$



$$y_1 + y_2 + \dots + y_n = \sum y = na + b\sum x_i \rightarrow (*)$$

$$\therefore \sum x_i y_i = a \sum x_i + b \sum x_i^2 \rightarrow (**)$$

dividing * by N

$$\mu_y = a + b\mu_x$$

$$a = \frac{\sum y_i}{N} - b \frac{\sum x_i}{N}$$

$$\sum x_i y_i = \frac{\sum x_i \sum y_i}{N} - b \frac{(\sum x_i)^2}{N} + b \sum x_i^2$$

$$\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{N} = b \left[\sum x_i^2 - \frac{(\sum x_i)^2}{N} \right]$$

$$\therefore a = \mu_y - b\mu_x$$

$$b = \frac{N \sum x_i y_i - (\sum x_i)(\sum y_i)}{N [\sum x_i^2] - (\sum x_i)^2}$$

show that the slope of the regression line

$$y = a + bx, \text{ is } r \frac{\sigma_y}{\sigma_x}$$

$$X = x - \mu_x; Y = y - \mu_y$$

$$r = \frac{\sum XY}{\sqrt{\sum X^2} \sqrt{\sum Y^2}}; \sigma_x = \sqrt{\frac{\sum (x - \mu_x)^2}{n}} = \sqrt{\frac{\sum X^2}{n}}; \sigma_y = \sqrt{\frac{\sum Y^2}{n}}$$

$$y = a + bx \rightarrow \sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

$$\mu_y = a + b\mu_x$$

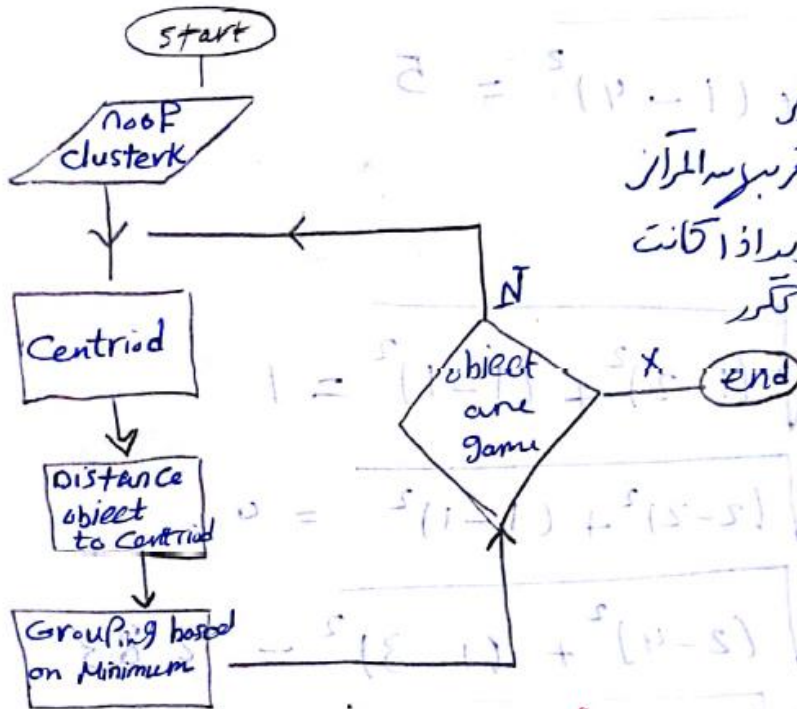
$$\sum XY = \sum (x - \mu_x)(y - \mu_y) = \sum (x - \mu_x)(a + bx - \mu_y) = \sum (x - \mu_x)(a + bx - a - b\mu_x)$$

$$b \sum (x - \mu_x)^2 = b \sum X^2$$

$$r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})} = \frac{b \sum X^2}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})} = \frac{b(\sqrt{\sum X^2})}{(\sqrt{\sum Y^2})} * \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \frac{b \left(\frac{\sqrt{\sum X^2}}{\sqrt{n}} \right)}{\left(\frac{\sqrt{\sum Y^2}}{\sqrt{n}} \right)} = \frac{b \sigma_x}{\sigma_y}$$

$$\therefore b = \text{slope} = \frac{r \sigma_y}{\sigma_x}$$

K - mean clustering



- ١- تحديد عدد التجمعات
- ٢- اختيار مراكز التجمعات
- ٣- تقسيم البيانات إلى مجموعات قريبة من المراكز
- ٤- حساب المراكز للتقسيم الجديد إذا كانت هذه القيم تتغير وإذا لم تتغير تكرر

$$d(C_1 A) = \sqrt{(-)^2 + (-)^2} ; d(C_2 A) = \sqrt{(-)^2 + (-)^2}$$

$$d(C_1 B) = \sqrt{(-)^2 + (-)^2} ; d(C_2 B) = \sqrt{(-)^2 + (-)^2}$$

$$d(C_1 C) = \sqrt{(-)^2 + (-)^2} ; d(C_2 C) = \sqrt{(-)^2 + (-)^2}$$

$$D_0 = \begin{bmatrix} S & S & S \end{bmatrix} \therefore G_1 = \{A\} ; G_2 = \{B, C\}$$

$$\text{new centroids } C_{11} (x_A, y_A) ; C_{12} = \left(\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2} \right)$$

REPEAT

Probabilities

- 1- التجربة العشوائية : التجربة الغير معروف نتائجها Random Experiment
- 2- Sample Space (S) : فضاء العينة هو الفضاء الذي يحتوي جميع نواتج التجربة العشوائية
- 3- Event : هو حدث يحدث داخل التجربة العشوائية
- 4- Probability of Event $P(A) = \frac{\text{no of A elements}}{\text{no of S elements}}$

Probability axioms الفروض الاحتمالية

$$1 - 0 \leq P(A) \leq 1$$

$$2 - P(S) = 1$$

3 – let A, B are mutually exclusive even $A \cap B = \phi$
 $\therefore P(A \cup B) = P(A) + P(B)$

Basic Probability Theory

- 1) $P(\phi) = 0$
- 2) $P(A^c) = 1 - P(A)$
- 3) $A \leq B \rightarrow P(A) \leq P(B)$
- 4) $P(A - B) = P(A) - P(A \cap B)$
- 5) $P(A \cup B) = P(A + B) = P(A) + P(B) - P(A \cap B)$

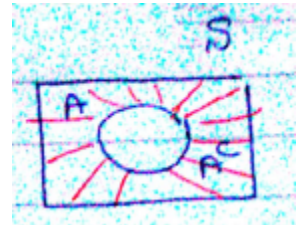
Proofs

$$1) P(\phi) = 0$$

$$\begin{aligned} A &= A \cup \phi \\ P(A) &= P(A \cup \phi) \\ P(A) &= P(A) + P(\phi) \text{ then } P(\phi) = 0 \end{aligned}$$

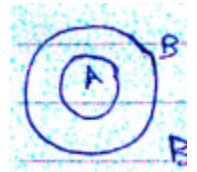
$$2) P(A^c) = 1 - P(A)$$

$$\begin{aligned} S &= A \cup A^c \\ A^c \cap A &= \phi \\ P(S) &= P(A \cup A^c) \\ 1 &= P(A) + P(A^c) \\ \therefore P(A^c) &= 1 - P(A) \end{aligned}$$



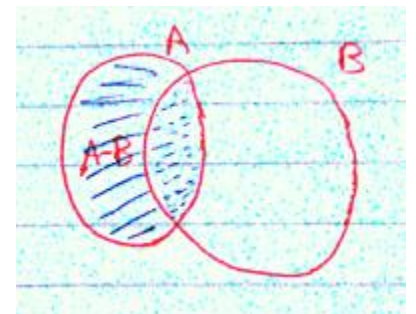
$$3) A \leq B \rightarrow P(A) \leq P(B)$$

$$\begin{aligned} P(A) &\leq P(B) \\ B &= (B - A) \cup A \\ (B - A) \cap A &= \phi \\ P(B) &= P((B - A) \cup A) \\ &\quad \text{+ve > 0} \\ P(B) &= \overbrace{P(B - A)}^{+ve > 0} + P(A) \\ \therefore P(B) &\geq P(A) \end{aligned}$$



$$4) P(A - B) = P(A) - P(A \cap B)$$

$$\begin{aligned} A &= (A - B) \cup (A \cap B) \\ (A - B) \cap (A \cap B) &= \phi \\ P(A) &= P(A - B) + P(A \cap B) \\ P(A - B) &= P(A) - P(A \cap B) \end{aligned}$$



$$5) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

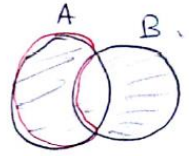
$$A \cup B = (A - B) \cup B$$

$$(A - B) \cap B = \phi$$

$$P(A \cup B) = P(A - B) + P(B)$$

$$= P(A) - P(A \cap B) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



6) if A, B are independent events then

1 - A^c, B are independent

2 - A, B^c are independent

3 - A^c, B^c are independent

$$\text{independency } P(A \cap B) = P(A)P(B)$$

$$(1) P(A^c \cap B) = P(A^c)P(B)$$

$$B - A = A^c \cap B$$

$$P(B - A) = P(B) - P(A \cap B)$$

$$\begin{aligned} P(A^c \cap B) &= P(B) - P(A)P(B) \\ &= P(B)(1 - P(A)) = P(B)P(A^c) \end{aligned}$$

$\therefore A^c, B$ are independent

$$(2) P(A \cap B^c) = P(A)P(B^c)$$

$$A \cap B^c = A - B$$

$$P(A - B) = P(A) - P(A \cap B)$$

$$P(A \cap B^c) = P(A) - P(A)P(B)$$

$$= P(A)[1 - P(B)] = P(A)P(B^c)$$

A, B^c independent

$$(3) P(A^c \cap B^c) = P(A^c)P(B^c)$$

$$(A \cup B)^c = A^c \cap B^c$$

$$P(A^c \cap B^c) = P[(A \cup B)^c] = 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= (1 - P(A)) - P(B) + P(A)P(B)$$

$$= (1 - P(A)) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B)) = P(A^c)P(B^c)$$

A^c, B^c independent

$A \text{ or } B (A + B)$	$A \cup B = P(A) + P(B) - P(A \cap B)$
$A \text{ and } B (A \times B)$	$A \cap B = P(A) * P(B)$
$B \text{ only}$	$B \cap A^c \cap C^c$
$A \text{ depended on } B \text{ and conditional}$	$P(A B) = \frac{P(A \cap B)}{P(B)}$
$A \text{ independent on } B \text{ and conditional}$ $\therefore A, B, A^c B^c \text{ are all independent}$	$P(A B) = P(A)$ $P(B A) = P(B)$ $P(A \cap B) = P(A)P(B)$
$A \cap B^c$	$A - B$
$(A \cup B)^c$	$A^c \cap B^c$
$P(A^c \cap B^c)$	$1 - P(A \cup B)$

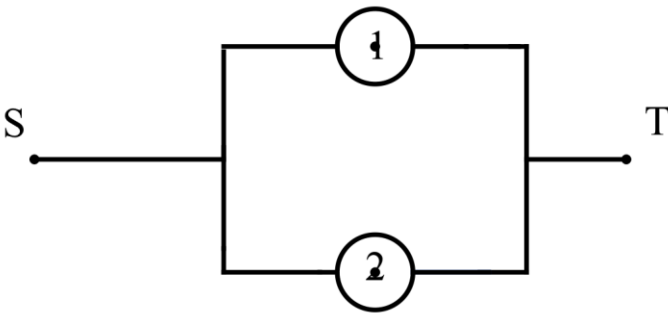
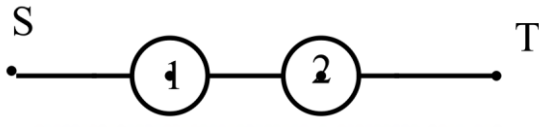
Methods to set probability space

- 1- Tree
- 2- Crossed Lines
- 3- $(\text{Possible Events})^{(\text{Repeations})}$

CaeSar Cipher

$$P = (C - K) \bmod 26$$

$$C = (P + K) \bmod 26$$

Parallel	Series																																																		
																																																			
$P = 1 + 2$	$P = 1 * 2$																																																		
<table><tr><th>E</th><th>1</th><th>2</th><th>1+2</th><th></th></tr><tr><td>E_1</td><td>0</td><td>0</td><td>0</td><td></td></tr><tr><td>E_2</td><td>0</td><td>1</td><td>1</td><td></td></tr><tr><td>E_3</td><td>1</td><td>0</td><td>1</td><td></td></tr><tr><td>E_4</td><td>1</td><td>1</td><td>1</td><td></td></tr></table> <p>$S = \{ E_1, E_2, E_3, E_4 \}$</p> <p>$P(A)(s \text{ to connect to } T)$ $P(A) = P(0,1) + P(1,0) + P(1,1)$</p> <p>عدد الواحد في الايفنت الواحد يترفع اس لاحتتماليه الفتح وعدد الاصفار يترفع اس لاحتتماليه عدم الفتح عدد مرات تكرار نفس عدد الواحد يبقي اضرب $P(A) = 2 * (0.8)^1 (0.2)^1 + 1 * (0.8)^2$</p>	E	1	2	1+2		E_1	0	0	0		E_2	0	1	1		E_3	1	0	1		E_4	1	1	1		<table><tr><th>E</th><th>1</th><th>2</th><th>1*2</th><th></th></tr><tr><td>E_1</td><td>0</td><td>0</td><td>0</td><td></td></tr><tr><td>E_2</td><td>0</td><td>1</td><td>0</td><td></td></tr><tr><td>E_3</td><td>1</td><td>0</td><td>0</td><td></td></tr><tr><td>E_4</td><td>1</td><td>1</td><td>1</td><td></td></tr></table> <p>$S = \{ E_1, E_2, E_3, E_4 \}$</p> <p>$P(A) = P(1,1)$ $P(A) = 1 * (0.8)^2$</p>	E	1	2	1*2		E_1	0	0	0		E_2	0	1	0		E_3	1	0	0		E_4	1	1	1	
E	1	2	1+2																																																
E_1	0	0	0																																																
E_2	0	1	1																																																
E_3	1	0	1																																																
E_4	1	1	1																																																
E	1	2	1*2																																																
E_1	0	0	0																																																
E_2	0	1	0																																																
E_3	1	0	0																																																
E_4	1	1	1																																																

R-step Experiment

If R is experiment has r-step to process

1st step has n_1 ways to process

2nd has n_2 ways

.

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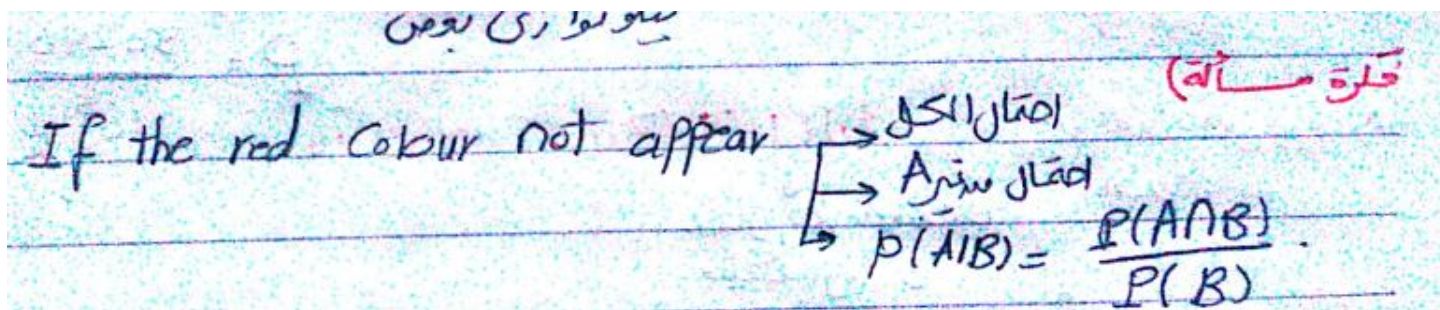
.

rth step has n_r ways to process

The totla number of ways to process $R = n_1 n_2 n_3 \dots n_r$

مثال سريع عاوز اعمل من 3 الوان علم افقي او راسي من 4 سترايب بس ميكنش نفس اللون في اتنين جنب بعض
الحل

$$P(A) = (3)(2)(2)(2) + (3)(2)(2)(2)$$



Permutation and Combinations

Combination:

no of ways to choose r from n without caring about order

$$C_r^n = \frac{n!}{r!(n-r)!}$$

Permutation :

no of ways to choose r from n with order

$$P_r^n = \frac{n!}{(n-r)!}$$

Ex : whats the minimum no of letters to make up 720 words

$$P_3^x$$

$$\text{solution : } \frac{\sqrt{720}}{3} = 9$$

$$9 * 10 * 11 \neq 720$$

$$7 * 8 * 9 \neq 720$$

$$10 * 9 * 8 = 720 \quad \therefore n = 10 \rightarrow P_3^{10} = 720 \rightarrow 10 \text{ letters}$$

$$\text{to be from letter } (A, B, C, D) \rightarrow P = \frac{P_3^4}{P_3^{10}} = \frac{1}{30}$$

Number of ways in grouping data in classes

After grouping data in classes

Number of ways of partition n distinct objects into groups containing n_1, n_2, \dots, n_k

$$= \frac{n!}{n_1! n_2! \dots n_k!}$$

where $n = n_1 + n_2 + \dots + n_k$

Proof

Show that the number of ways of the partition object into groups containing n_1, n_2, \dots, n_k objects is $\frac{n!}{n_1! n_2! \dots n_k!}$

We can partition n objects into groups by k steps

- First step : shows n_1 from n to put it in $G_1 \rightarrow C_{n_1}^n$
- Second step : shows n_2 from $(n - n_1)$ to put it in $G_1 \rightarrow C_{n_2}^{n-n_1}$
- .
- .
- .
- K step : shows n_k from $n - n_1 - n_2 - \dots - n_{k-1}$ to put it in $G_1 \rightarrow C_{n_k}^{n-n_1-n_2-\dots-n_{k-1}}$
- Then, total number = $C_{n_1}^n * C_{n_2}^{n-n_1} * \dots * C_{n_k}^{n-n_1-n_2-\dots-n_{k-1}}$
- $\frac{n!}{n!(n-n_1)!} * \frac{(n-n_1)!}{n!(n-n_1-n_2)!} * \dots * \frac{(n-n_1-n_2-\dots-n_{k-1})!}{n_k!(n-n_1-n_2-\dots-n_{k-1})!} = \frac{n!}{n_1! n_2! \dots n_k!}$

Total Probability

If sample space S partitions by A_1, A_2, \dots, A_n where $S = A_1 \cup A_2 \dots \cup A_n$ and $A_i \cap A_j = \phi$ and $B \subset S$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots + P(B|A_n)P(A_n)$$

Proofs

Show that if A_1, A_2, \dots, A_n partitions of S and $B \subset S \therefore P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$

$$\therefore B = (B \cap A_1) \cup (B \cap A_2) \cup \dots (B \cap A_n)$$

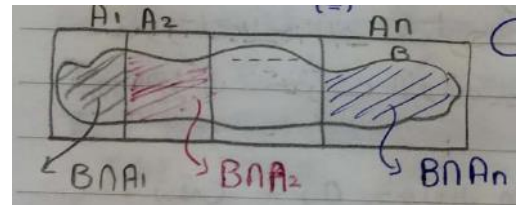
$$\because A_i \cap A_j = \phi$$

$$\therefore P(B) = P(B \cap A_1) + P(B \cap A_2) \dots + P(B \cap A_n) = \sum_{i=1}^n P(B \cap A_i)$$

$$\because P(B|A_i) = \frac{P(B \cap A_i)}{P(A_i)}$$

$$\therefore P(B|A_i)P(A_i) = P(B \cap A_i)$$

$$\therefore P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$$



Show that $P(A_k|B) = \frac{P(B|A_k)P(A_k)}{(P(B|A_1)P(A_1)+P(B|A_2)P(A_2)+\dots+P(B|A_n)P(A_n))}$ where $S = A_i \cup A_j$, $A_i \cap A_j = \phi$

From total probability $P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$

$$P(A_k|B) = \frac{P(B \cap A_k)}{P(B)}, P(B|A_k) = \frac{P(B \cap A_k)}{P(A_k)}$$

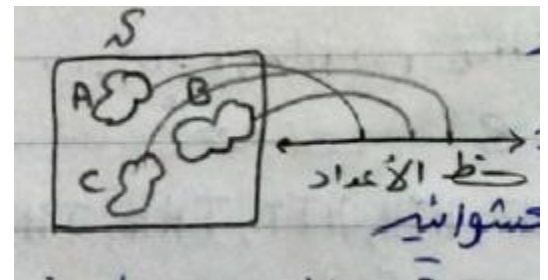
$$\therefore P(B \cap A_k) = P(A_k)P(B|A_k)$$

$$\therefore P(A_k|B) = \frac{P(B \cap A_k)}{P(B)} = \frac{P(A_k)P(B|A_k)}{\sum_{i=1}^n P(B|A_i)P(A_i)} \quad \text{Bay's Theorem}$$

Random Variables

We move the problem from words or events to numbers

- 1- Can be discrete (countable)
- 2- Continuous



Types of Random Variable

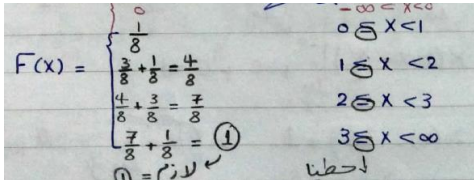
Discrete RV	Continuous RV
Finite number (can be limited in a table)	Much more we used periods $a \leq x \leq b$

Note :

$f(x) \rightarrow$ Probability fun

$F(x) \rightarrow$ C.D.F cumulative

(1) Probability function f(x)

Probability Density function P.D.F	Probability Mass Function P.M.F						
<p>Continuous Random variables (C.R.V)</p> <p>Probability density function</p> <p><u>CONDITIONS</u></p> <p>(1) $f(x_i) \geq 0$</p> <p>(2) $\int_{-\infty}^{\infty} f(x)dx = 1$</p> <p>Periods</p> 	<p>Discrete Random Variables (D.R.V)</p> <p>Probability mass function</p> <p><u>CONDITIONS</u></p> <p>(1) $f(x_i) \geq 0$</p> <p>(2) $\sum_{i=1}^n f(x_i) = 1$</p> <p>Can be in a table</p> <table><tr><td>X</td><td></td><td></td></tr><tr><td>P(x)</td><td></td><td></td></tr></table>	X			P(x)		
X							
P(x)							

(2) Cumulative Distribution Function F(x)

Continuous	Discrete
$F(x) = \int_{-\infty}^x f(x)dx$ <p>(1) $F(-\infty) = 0$, $F(\infty) = 1$ (2) $f(x) = \frac{dF(x)}{dx}$ (3) $P(a \leq x \leq b) = F(b) - F(a)$</p>	$F(x) = \sum_{-\infty}^x f(x)$ <p>(1) $F(-\infty) = 0$, $F(\infty) = 1$ (2) $f(x_i) = F(x_i) - F(x_i - 1)$ $P(a \leq x \leq b) = F(b) - F(a)$</p>

(3) The Expectation [E(x)] التوقع

إذا تحولت التجربة العشوائية الى خط الاعداد فان توقع الحد x يمكن حسابه

$E(x) = \mu$	
C.R.V	D.R.V
$E(x) = \int_{-\infty}^{\infty} x f(x)dx$	$E(x) = \sum x f(x)$

$E(g(x))$	
C.R.V	D.R.V
$E(g(x)) = \int_{-\infty}^{\infty} g(x) f(x)dx$	$E(x) = \sum g(x_i) f(x_i)$ <p>نحسب الـ $g(x)$ عند كل خانه في الـ x ونضربها في الداله ونجمع</p>

في الاعداد ذات الاحتمالات المتساويه - التوقع هو المتوسط الحسابي μ

Properties of Expectation

$$(1) E(a) = a \quad (2) E(ax + by) = aE(x) + bE(y)$$

$$(3) E(ax + b) = aE(x) + b$$

If x is C. R. V show that $E(ax + b) = aE(x) + b$ (هام)

$$E(\text{اي حاجه}) = \int_{-\infty}^{\infty} \text{اي حاجه} f(x)dx$$

$$\therefore E(ax + b) = \int_{-\infty}^{\infty} (ax + b)f(x)dx$$

$$= a \int_{-\infty}^{\infty} x f(x)dx + b \int_{-\infty}^{\infty} \widetilde{f(x)}^{P.D.F} dx = aE(x) + b(1)$$

(4) The Variance and standard deviation التباين والانحراف المعياري

هو مقدار البعد او القرب من التوقع لقيم المتغير

$$\sigma = \sqrt{V(x)}$$

$$V(x) = \sigma^2 = E(x^2) - (E(x))^2$$

$V(x)$	
C.R.V	D.R.V
$V(x) = E((x - \mu)^2)$	$V(x) = E[(x - \mu)^2]$
$V(x) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$	$V(x) = \sum (x - \mu)^2 f(x)$

Properties of Variance

- (1) $V(ax) = |a|^2 V(x)$ (2) $V(ax + by) = a^2 V(x) + b^2 V(y)$
 (3) $V(ax + b) = a^2 V(x)$

if x is CRV show that $V(x) = E(x^2) - (E(x))^2$

$$V(x) = E((x - \mu)^2) \quad \& \quad E(x) = \mu$$

$$V(x) = E(x^2 - 2\mu x + \mu^2) = E(x^2) - 2E(x) * E(x) + E(x)^2 = E(x^2) - (E(x))^2$$

show that $V(ax + b) = a^2 V(x)$

$$V(\text{اي حاجه}) = [E(\text{اي حاجه})^2 - (E(\text{اي حاجه}))^2]$$

$$\therefore V(ax + b) = [E(ax + b)^2 - (E(ax + b))^2]$$

$$\begin{aligned} &= E(a^2 x^2 + 2abx + b^2) - [a(E(x)) + b]^2 \\ &= a^2 E(x^2) + 2abE(x) + b^2 - [a^2(E(x))^2 + 2abE(x) + b^2] \\ &= a^2 [E(x^2) - (E(x))^2] = a^2 V(x) \end{aligned}$$

(5) The r^{th} moment العزم الرائي

About mean	About origin
μ_r	μ'_r
$\mu_r = E[(x - \mu)^r]$	$\mu'_r = E(x^r)$
$\mu_0 = 1$	$\mu'_0 = E(x^0) = E(1) = 1$
$\mu_1 = E(x - \mu) = 0$	$\mu'_1 = E(x) = \mu$
$\mu_2 = E(x - \mu)^2 = V(x)$	$\mu'_2 = E(x^2)$
	$V(x) = \mu'_2 - (\mu'_1)^2$

$$\mu_2 = \mu'_2 - (\mu'_1)^2$$

(6) The Moment Generating Function

$M_x(t) = E(e^{xt})$	
<p>C.R.V</p> $M_x(t) = \int_{-\infty}^{\infty} e^{xt} f(x) dx$	<p>D.R.V</p> $M_x(t) = \sum e^{xt} f(x)$

Properties of $M_x(t)$

$$(1) \mu'_r = E(x^r) = \frac{d^r M_x(t)}{dt^r} \quad @t = 0$$

$$E(x) = M'_x(0), E(x^2) = M''_x(0) \dots E(x^n) = M^n_x(0)$$

$$(2) M_{x+a}(t) = e^{at} M_x(t)$$

$$(3) M_{bx}(t) = M_x(bt)$$

(4) *generalization*

$$M_{bx+a} = e^{at} M_x(bt)$$

show that $M_{x+a} = e^{at} M_x(t)$ and $M_{bx}(t) = M_x(bt)$

$$M_{\text{اي حاجه}}(t) = E\left(e^{(\text{اي حاجه})t}\right)$$

$$M_{x+a}(t) = E\left(e^{(x+a)t}\right) = E\left(e^{at} e^{xt}\right) = e^{at} E\left(e^{xt}\right)$$

$$M_{bx}(t) = E\left(e^{bxt}\right) = E\left(e^{x(bt)}\right) = M_x(bt)$$

Probability Distribution

[1] Discrete probability distribution

An experiment will have two events their probability are **P and Q** and test is repeated **n times** so we know the type of distribution

name	n	Dist f(x)	μ or $E(x)$	$V(x)$	$M_x(t)$	Notes
Bernoulli	$n=1$	$p^x q^{1-x}$ $x = 0, 1$	p	pq	$q + pe^t$	عدد مرات التجربة n كل مره يجرب بها التجربه هناك احتمال من اثنين q(fail) او p(success)
Binomial $b(x; n, p)$	$n \leq 50$	$C_x^n p^x q^{n-x}$ $x = 0, 1, \dots, n$	np	npq	$(q + pe^t)^n$	In single trial $p+q=1$ x = number pf success
Poisson $P(x; \lambda)$	$n > 50$	$\frac{e^{-\lambda} \lambda^x}{x!}$ $\lambda = np$ $x = 0, 1, \dots, n$	λ	λ	$e^{\lambda(e^t-1)}$	
-ve Binomial $f(x; l, p)$	Unknown	$C_{k-1}^{x-1} p^k q^{x-k}$ $x = k, k+1, \dots$	$\frac{k}{p}$	$\frac{kq}{p^2}$		X : number of trial until r^{th} success is observed
Geometric $G(x)$	Unknow	pq^{x-1} $x = 1, 2, 3, \dots$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1 - qe^t}$	<u>Special case of Geom.</u> $r=1$ x : no of trial until first success is observed

Proofs

Find the value of $E(x), V(x), M_x(t)$ for Bernolli distribution

X	0	1
P(X)	q	p

$$E(x) = \sum x f(x) = (0)(q) + (1)p = p$$

$$V(x) = E(x^2) - (E(x))^2 = (0^2)(q) + (1^2)(p) - p^2 = p(1 - p) = pq$$

$$M_x(t) = E(e^{xt}) = \sum e^{x_i t} f(x_i) = e^{0t} q + e^{1t} p = q + pe^t$$

Binomial

مفتاح الاختصارات في الحل هو مفكوك ذات الحدين

$$(a + b)^n = \sum_{x=0}^n C_x^n a^x b^{n-x}$$

يمكن اجيب المعاملات من باسكال

Find the value of $E(x), V(x)$ and $M_x(T)$ for Binomial Distribution

$$E(x) = \sum x f(x) = \sum x C_x^n p^x q^{n-x}$$

دائما نحاول تطير x عشان اللي جوا التجميع يرجع قوس عن طريق فك التوفيق Combination و الاختصار معه

واحاول اعدل التجميع غشان اطلع صورته ذات الحدين اللي فوق دي واقوم اشيل التجميع واحط القوس

$$E(x) = \sum x \frac{n!}{x!(n-x)!} p^x q^{n-x} = \sum \frac{n!}{(x-1)!(n-x)!} p^x q^{n-x}$$

$$y = x - 1 \rightarrow x = y + 1$$

$$\begin{aligned} E(x) &= \sum \frac{n!}{y!(n-y-1)!} p^{y+1} q^{n-y-1} = p \sum \frac{n(n-1)!}{y!((n-1)-y)!} p^y q^{(n-1)-y} \\ &= np \sum \frac{(n-1)!}{y!((n-1)-y)!} p^y q^{(n-1)-y} = np \sum_0^n C_y^{n-1} p^y q^{(n-1)-y} = np(p+q)^{n-1} = np * 1 = np \end{aligned}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x(x-1)) = E(x^2 - x) = E(x^2) - E(x)$$

$$\therefore E(x^2) = E(x(x-1)) + E(x) \rightarrow (1)$$

$$E(x(x-1)) = \sum x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum \frac{n!}{(x-2)!(n-x)!} p^x q^{n-x}$$

$$\text{let } y(x-2) \rightarrow x = y + 2$$

$$= \sum \frac{n!}{y!((n-2)-y)!} p^y p^2 q^{(n-2)-y}$$

$$= n(n-1)p^2 \sum \frac{(n-2)!}{y!((n-2)-y)!} p^y q^{(n-2)-y} = n(n-1)p^2 \sum C_y^{n-2} p^y q^{(n-2)-y}$$

$$= n(n-1)p^2(p+q)^{n-2}$$

$$\therefore E(x(x-1)) = n(n-1)p^2$$

$$\therefore E(x)^2 = E(x(x-1)) + E(x) = n(n-1)p^2 + np$$

$$\therefore V(x) = E(x^2) - (E(x))^2 = n^2 p^2 - n(n-1)p^2 + np = np(1-p) = npq$$

$$M_x(t) = \sum e^{xt} f(x) = \sum e^{xt} C_x^n p^x q^{n-x}$$

$$M_x(t) = \sum C_x^n q^{n-x} (pe^t)^x = (pe^t + q)^n$$

Poisson

Find the value of $E(x)$, $V(x)$ and $M_x(T)$ for Poisson Distribution

$$E(x) = \sum x f(x) = \sum x \frac{\lambda^x}{x!} e^{-\lambda} = e^{-\lambda} \sum \frac{\lambda^x}{(x-1)!}$$

$$\text{let } y = x - 1 \rightarrow x = y + 1$$

$$\therefore E(x) = e^{-\lambda} \sum \frac{\lambda^y * \lambda}{y!} = e^{-\lambda} \lambda \sum \frac{\lambda^y}{y!} = e^{-\lambda} \lambda e^{\lambda}$$

$$\therefore E(x) = \lambda$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x(x-1)) = E(x^2) - E(x) \Rightarrow E(x^2) = E(x(1-x)) + E(x)$$

$$E(x(x-1)) = \sum x(x-1)f(x) = \sum x(x-1) \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\therefore = e^{-\lambda} \sum \frac{\lambda^x}{(x-2)!} \quad \text{let } y = x-2, \quad x = y+2$$

$$= e^{-\lambda} \sum \frac{\lambda^y \lambda^2}{y!} = \lambda^2 e^{-\lambda} e^{\lambda} = \lambda^2$$

$$\therefore E(x^2) = \lambda^2 + \lambda$$

$$\therefore V(x) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$\therefore V(x) = \lambda$$

$$M_x(t) = E(e^{xt}) = \sum e^{xt} f(x) = \sum e^{xt} \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum \frac{(e^t \lambda)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$$

Show that $\lim_{n \rightarrow \infty} b(n, x, p) = p(x, \lambda)$

Where

$b(n, x, p)$ is P.M.F of binomial distribution
 $P(x, \lambda)$ is P.M.F of poisson distribution

$$b(n, x, p) = C_x^n p^x q^{n-x}, \quad p(x, \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\lim_{n \rightarrow \infty} b(n, x, p) = \lim_{n \rightarrow \infty} \frac{n!}{x! (n-x)!} p^x q^{n-x}$$

$$\text{NOTES : } \lambda = np; \lim_{n \rightarrow \infty} \left(1 + \frac{q}{n}\right)^n = e^q$$

$$p = \frac{\lambda}{n}; q = 1 - p = 1 - \frac{\lambda}{n}$$

$$\lim_{n \rightarrow \infty} b(n, x, p) = \lim_{n \rightarrow \infty} \frac{n(n-1)(n-2) \dots (n-x-1)(n-x)! \lambda^x}{x! (n-x)!} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

وزع التفاضلية

$$= \frac{\lambda^x}{x!} \lim_{n \rightarrow \infty} \frac{n}{n} * \frac{n-1}{n} * \frac{n-2}{n} * \dots * \frac{n-x-1}{n} * \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n * \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{x+1}{n}\right) e^{-\lambda} \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} (1) e^{-\lambda} (1) = \frac{\lambda^x}{x!} e^{-\lambda} = p(x, \lambda)$$

Geometric

التجربة تنقسم لحدثين q و p و تتكرر عدد ∞ من المرات نستخدم التوزيع الهندسي

احتمال ظهور الحدث الذي احتماله بعد x من المرات قبلها ظهوره q من المرات

(Remarks)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \dots = \sum_{n=1}^{\infty} nx^{n-1}$$

$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + \dots = \sum_{n=0}^{\infty} nx^n$$

$$e^{f(x)} = \sum_{n=0}^{\infty} \frac{(f(x))^n}{n!}$$

$$\frac{d}{dx} \left(\frac{x}{(1-x)^2} \right) = \frac{(1-x)^2(1) + 2x(1-x)}{(1-x)^4} = \frac{1+x}{(x-1)^3}$$

$$\frac{1+x}{(1-x)^3} = 1 + 4x + 9x^2 + \dots = \sum_{n=0}^{\infty} n^2 x^{n-1}$$

Find the value of $E(x)$, $V(x)$ and $M_x(T)$ for Geometric Distribution, $F(x) = pq^{x-1}$, $x = 1, 2, \dots$

$$E(x) = \sum x f(X) = \sum_{x=1}^{\infty} x p q^{x-1} = p \sum_{x=1}^{\infty} x q^{x-1}$$

$$\begin{aligned} E(x) &= p [1 + 2q + 3q^2 + 4q^3 + \dots] \\ &= \frac{p}{q} [q + 2q^2 + 3q^3 + \dots] \\ &= \frac{p}{q} \left[\frac{q}{(1-q)^2} \right], p + q = 1 \rightarrow \frac{p}{q} \left[\frac{q}{p^2} \right] = \frac{1}{q} \left[\frac{q}{p} \right] = \frac{1}{p} \end{aligned}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum x^2 f(x) = \sum x^2 p q^{x-1} = p \sum x^2 q^{x-1} = p \left[\frac{1+q}{(1-q)^3} \right] = p \left[\frac{1+q}{(p)^3} \right] = \left[\frac{1+q}{(p)^2} \right]$$

$$\therefore V(x) = \frac{1+q}{p^2} - \frac{1}{p^2} = \frac{1+q-1}{p^2} = \frac{q}{p^2}$$

$$V(x) = \frac{q}{p^2}$$

$$M_{x(t)} = E(e^{xt}) = \sum e^{xt} f(x) = \sum e^{xt} p q^{x-1}$$

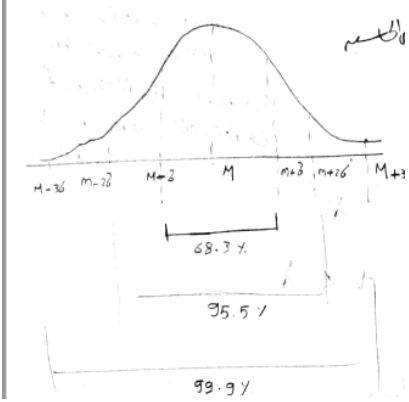
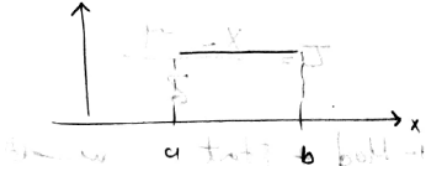
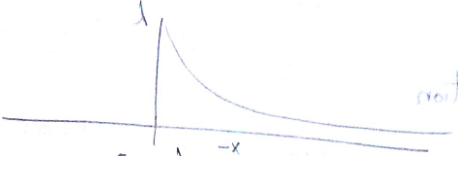
$$= pq^{-1} \sum_{x=1}^{\infty} (e^t q)^x = e^t q + e^{2t} q^2 + \dots$$

ناقصها 1 عشان تبقى شبه الصور

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$$

$$= \frac{p}{q} [-1 + 1 + qe^t + (qe^t)^2 + (qe^t)^3 + \dots] = \frac{p}{q} \left[-1 + \frac{1}{1-qe^t} \right] = \frac{p}{q} \left[\frac{-1 + qe^t + 1}{1-qe^t} \right] = \frac{pe^t}{1-qe^t}$$

[2] Continuous probability distribution

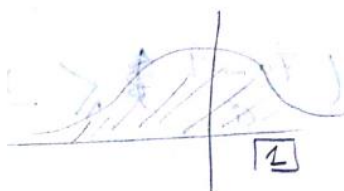
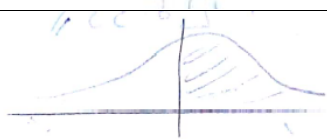
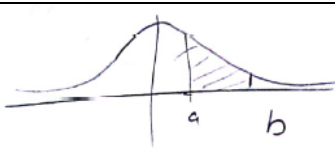
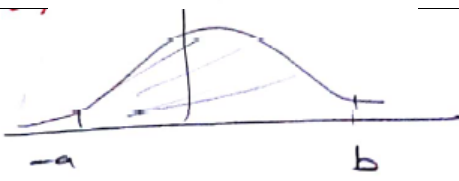
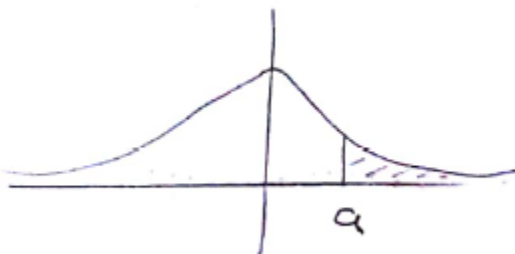
Normal Distribution	Uniform Distribution	Exponential Distribution
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $E(x) = \mu$ $V(x) = \sigma^2$ $M_x(t) = e^{\mu t + \frac{t^2 \sigma^2}{2}}$  <p>Standard Normal Distribution</p> $\mu = 0, \quad \sigma^2 = 1$ $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2}$ $Z = \frac{x - \mu}{\sigma}$ <p>عند دراسة الخواص عند تماثل البيانات حول المتوسط</p> <p>نستخدم الآلة الحاسبة</p> <p>Mode -> stat -> AC -> shift+1 -> 5-dist -> Q!</p> <p>تحسب من أول الصفر لحد النقطة بتاعتي</p> <p>مجموع من -∞ الي ∞ = 1</p> <p>مجموع من 0 الي ∞ = 0.5</p>	$f(x) = \begin{cases} \frac{1}{b-a} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$ $E(x) = \mu = \frac{a+b}{2}$ $V(x) = \sigma^2 = \frac{(b-a)^2}{12}$ $M_x(t) = \frac{(e^{bt} - e^{at})}{(b-a)t}$  <p>يمثل احتمال عمل system بانتظام في فترة زمنية محدد او زمن معين و يمثل احتمال وصول سياره او مكالمه تليفونية ف زمن معين</p>	$f(x) = \begin{cases} \lambda e^{-x} & ; x > 0 \\ 0 & ; x < 0 \end{cases}$ $E(x) = \mu = \frac{1}{\lambda}$ $V(x) = \sigma^2 = \frac{1}{\lambda^2}$ $M_x(t) = \frac{\lambda}{\lambda - t} \quad t < \lambda$  <p>يستخدم في قياس احتمالات اعمار الاجهزه الكهربيه وتستخدم في حساب الوقت اللازم في تعطل بعض الانظمة الكهربيه كما تستخدم لحساب الوقت اللازم لوقوع حدث ما</p>

Normal Distribution

Steps of solving with calculator :

- 1- Convert it to standard $\tau = \frac{x-\mu}{\sigma}$
- 2- *Mode* \rightarrow *stat* \rightarrow *AC* \rightarrow *shift* + 1 \rightarrow *Distrib* \rightarrow **Q**(

Properties of Normal Distribution

1	$P(-\infty < \tau < \infty) = 1$	
2	$P(0 < \tau < \infty) = 0.5$	
3	$P(a \leq \tau \leq b) = Q(b) - Q(a)$ a, b are +ve	
4	$P(-a \leq \tau \leq b) = Q(b) + Q(a)$	
5	$P(\tau \geq a) = 0.5 - Q(a)$	

show that the prob fn for Normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ is P.D.F

To be PDF We need to prove

$$(1) f(x) \geq 0 \quad (2) \int_{-\infty}^{\infty} f(x) dx = 1$$

to prove

$$(1) f(x) \geq 0 \quad \text{since } e^{-\infty} = 0$$

$$(2) \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$\text{let } y = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + \sigma y \Rightarrow dx = \sigma dy$$

$$I = \frac{\sigma}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy \rightarrow (*)$$

$$I_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy$$

To Polar

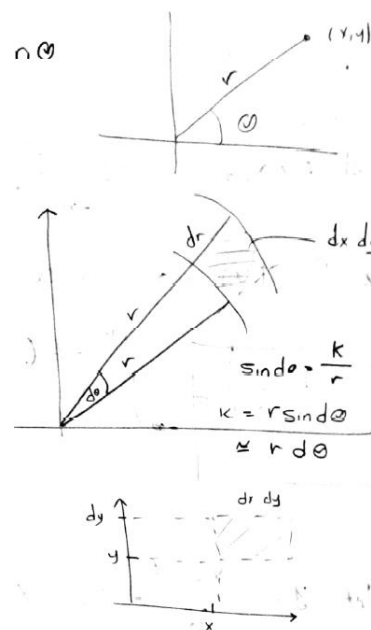
$$I_1^2 = I_1 I_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy * \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx$$

$$I_1 = \iint_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+y^2)} dx dy$$

$$x = r \cos \theta ; y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$dxdy = r dr d\theta$$



$$\begin{aligned} \therefore I_1^2 &= \int_0^{2\pi} \left(\int_0^{\infty} r e^{-\frac{1}{2}r^2} dr \right) d\theta \\ &= \int_0^{2\pi} \left(\left[e^{-\frac{1}{2}r^2} \right]_0^{\infty} \right) d\theta \\ &= - \int_0^{2\pi} [0 - 1] d\theta = [\theta]_0^{2\pi} = 2\pi \end{aligned}$$

$$I_1 = \sqrt{2\pi} \quad \text{substitute on } (*)$$

$$I = \frac{1}{\sqrt{2\pi}} * \sqrt{2\pi} = 1$$

Find $E(x)$, $V(x)$ and $M_x(t)$ for Normal distribution if $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

We find $M_x(t)$ then we try to get $E(x)$ and $V(x)$ from it

$$E(x) = \left[\frac{d}{dt} M_x(t) \right]_{t=0} ; \quad E(x^2) = \left[\frac{d^2 M_x(t)}{dt^2} \right]_{t=0}$$

$$M_x(t) = E(e^{xt}) = \int_{-\infty}^{\infty} e^{xt} * \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\left[xt - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]} dx$$

$$xt - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 = -\frac{1}{2\sigma^2} [(x-\mu)^2 - 2\sigma^2 xt]$$

$$= -\frac{1}{2\sigma^2} [x^2 - 2\mu x + \mu^2 - 2\sigma^2 tx]$$

$$-\frac{1}{2\sigma^2} [x^2 - 2(\mu + \sigma^2 t)x + \mu^2]$$

نكمل المربع

$$\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \text{الباقى} \left(\frac{1}{2}\right)^2 \text{معامل الثانى (الاشارة بعده جنر الاول)}$$

$$= -\frac{1}{2\sigma^2} [(x - (\mu + \sigma^2 t))^2 - (\mu + \sigma^2 t)^2 + \mu^2]$$

$$= -\frac{1}{2\sigma^2} [(x - (\mu + \sigma^2 t))^2 - 2\sigma^2 t\mu - \sigma^4 t^2]$$

نعوض في M_x و نطلع اللي مفهوش x برا التكامل

$$M_x(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(-2\sigma^2 t\mu - \sigma^4 t^2)} * \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-(\mu+\sigma^2 t)}{\sigma}\right)^2} dx$$

$$I_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-(\mu+\sigma^2 t)}{\sigma}\right)^2} dx$$

$$Z = \frac{x - \mu - \sigma^2 t}{\sigma} \Rightarrow dx = \sigma dz$$

$$I_1 = \sigma \int_{-\infty}^{\infty} e^{-\frac{Z^2}{2}} dZ$$

$$I_1^2 = I_1 * I_1 = \sigma^2 \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} dy * \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$I_1^2 = \sigma^2 \iint_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+y^2)} dx dy$$

$$x=r\cos\theta\qquad y=r\sin\theta$$

$$dxdy=rdrd\,\theta$$

$$I_1^2=(2\pi)(\sigma^2)$$

$$\mathbf{M} \text{ بالتعويض في}$$

$$M_x(t)=e^{\mu t+\frac{1}{2}\sigma^2t^2}$$

$$E(x)=\left[\frac{d}{dt}\,M_x(t)\right]_{t=0}=\left[\left(\mu+\frac{1}{2}\sigma^2(2t)\right)e^{\mu t+\frac{1}{2}\sigma^2t^2}\right]_{t=0}=\mu$$

$$V(x)=E(x^2)-\big(E(x)\big)^2=\sigma^2+\mu^2\,-\mu^2=\sigma^2$$

$$as\;E(X)=\mu\;\;and\;E(x^2)=\left[\frac{d^2}{dt^2}\,M_x(t)\right]_{t=0}=\;\sigma^2+\mu^2$$